THE STUDY OF THE DYNAMICS OF VIBRATION OF A CANTILEVER UNDER LATERAL IMPACT OF AN ELASTIC LOAD. PART V (Experiment)

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ABSTRACT. Cantilevers of different materials but of same length and cross section are transversely struck at different points by hammers of different materials and masses, possessing different striking velocities. Photographic method of measurement is adopted. It is found that the duration of impact changes discontinuously with striking distance. Ordinarily, the duration of impact for particular struck point decreases in magnitude as the impinging velocity of the load increases. Soon after impact begins, certain points of the cantilever (other than the struck point) move for a short while along negative direction before it can acquire usual forward motion. Vibration ensues soon after impact begins The theory developed in parts I to III in this series of papers has been verified with the experimental results. The agreement between the theory and the experiment is remarkable

EXPERIMENTAL

Mason (1936), Arnold (1937) ando thers have reported some experiments on supported beams using electrical method of measurement Some experiments on cantilever were reported by Banerjee (1966). In this paper, the writer has exhaustively investigated different aspects of the problem systematically.

The experimental set-up is almost similar to that used by Banerjee (1966). The hammer of particular material and mass is released from a distance to ensure a desired impinging velocity as it perpendicularly strikes specified points of the cantilever (including the free end) at regular intervals of 5 cms. Light from carbon arc lamp, facing the narrow slit of a camera box vertically downwards, is obstructed by the pointer of the tuning fork and the point of contact of the hammer and the cantilever to cast their shadows on the running photographic paper, pinned on the photocarrier. The photocarrier slides inside the camera box.

In this way, the effects of the striking distance, 'mass ratio', striking velocity and materials of the cantilever and hammer, on the duration of impact as also on the displacment of the cantilever and hammer are studied. The results of the experiments may be found in different curves and a plate. The displacement of the cantilever at any point other than the struck point is also studied, (vide fig 8B). The slit in this case is brought under the point to be photographed with the arc lamp and the vibrating tuning fork above it.

To reproduce the experimental time-displacement curve for any point of the cantilever during and after impact, to any scale, the corresponding shadowgraph including the waves due to the tuning fork at the top is projected upon a squared paper with the help of an epidiascope. The curve traced out by the surface of the cantilever is drawn upon the paper. The number of waves due to the tuning fork of frequency 100 c.p.s. contained in the portion of the shadowgraph under study and lying within a known length of the time-axis gives the time scale. The depth of the shadow of the bar at rest, towards left of each shadowgraph (taken equivalent to 1.27 cms) helps to fix the displacement scale. For measuring the striking velocity, the method used in part IV (Banerjee, 1966), is adopted. The maximum arc along which the hammer swings to strike the cantilever is measured from the shadow of the particular outline of the hammer on a graduated translucent scale, placed very close and parallel to the path of the hammer. The length of the pendu- lum bob (hammer) and its arc of swing help calculate the impinging velocity.

PARTICULARS OF CANTILEVERS AND HAMMERS

Cantilever A, mild steel rod, length 95 cms, dia. 1.27 cms.

Cantilever B, brass rod, length 95 cms, dia. 1.27 cms.

Hammer C,spherical, mild steel, weight = 285.5 gms., radius at contact surface, 2 cms.

Hammer D, spherical, brass, weight = 285.5 gms., radius at contact surface, 2 cms.

Hammer E, spherical, brass, weight = 108.8 gms., radius at contact surface,, 1 cm.

The curves e_1 and e_2 of fig. 1. show variation of the duration of impact (first contact) with striking distances for the same mild steel cantilever A,

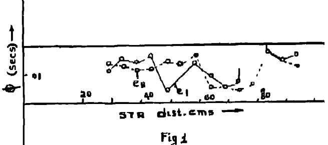
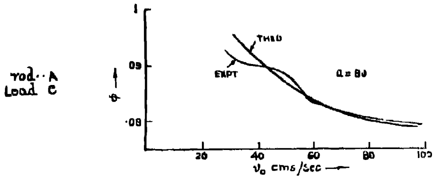


Fig 1

struck by mild steel hammer-C and brass hammer—D respectively. The hammers have equal weight and possess equal striking velocity of 65 cms. per sec. From these curves, we find that the duration of impact fluctuates discontinuously with striking distance. These curves differ due to a difference in the values of the elastic constant, (E_2) of the striking hammers. In most cases of striking distances, this difference is found to be very small, being measured in thousandths of a second, but for such striking distances as 50 and 80 cms. this difference is large. For striking distance of 80 cms. the durations of impact is .08284 sec and is not shown in the curve. The lower magnitude of the duration of impact in these particular cases indicates stimulation of *larger* number of modes into activity

The curves in fig. 2, each representing particular struck point, are drawn to show the variation of the duration of impact (first contact) with the striking



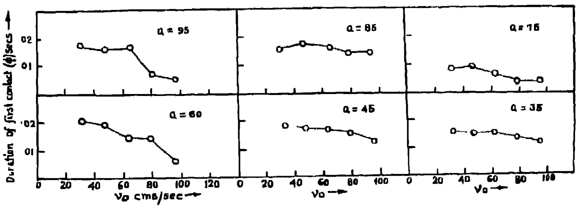


Fig. 2

velocity of the hammer. It is found that ordinarily the duration of impact first contact) changes with the striking velocity of the hammer. It is found that ordinarily the duration of impact slightly diminishes in magnitude as the striking velocity increases, and for striking distances such as 95 cms. and 60 cms, it appears that beyond a certain limiting velocity (at or around 79.25cms/sec for 95 cms., and 94.75 cms/sec for 60 cms), the duration of impact abruptly changes. This is due to the higher modes becoming effective at higher velocities

The curves in fig. 3 show the variation of total duration of impact (measured from the time the load comes in contact to the instant it completely leaves the region of vibration of the rod) with striking distance for different striking velocities of the hammer. It is found that the total duration of impact fluctuates as striking distance, increases, but maintaining the fluctuating character, the total duration of impact increases abruptly from about 55 cms. onward and attains the maximum value at or around 80 cms. of striking distance. Abrupt increases in the magnitude of the total duration of impact observed from 55 cms. show that, at the instant $P = 0$ (Banerjee, 1966 ; Part III) and for a specified number of modes being

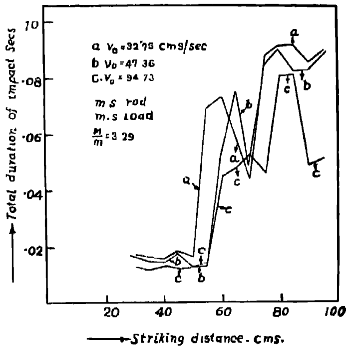
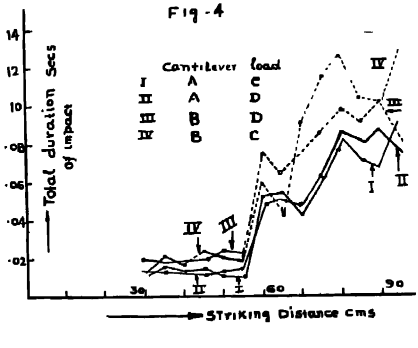


Fig 3

effective, $\Sigma A_s \cos q_s t$ (eqn. 13, Pt II) changes sign for a struck point near about 55cms and this change is effected with absorption of maximum energy by the beam (Banerjee, 1966, Pt. IV).

Fig. 4 shows the variation of the total duration of impact with the striking distance for two cantilevers of different materials but of same length and cross section, being struck by two hammers of different materials but of equal mass and possessing equal striking velocity of 65 cms /sec. It is found that the nature of fluctuation of total duration of impact with striking distance depends more or less on the material of the hammer.

Fig -4



The shadowgraphs (fig 8) show that the rod begins to vibrate as soon as impact begins. Usually almost for all striking distances, the phenomenon of 'multiple contacts' is observed. For certain struck points (such as 80 cms) the form of time-displacement curve appears to be more or less sinusoidal but for struck points, such as 47.5 cms this curve is more undulating showing importance of the second mode of vibration. Thuss timulation of particular number of modes into activity depends on the striking distance.

It is further found that for a short while from the instant impact begins, the motion of certain points other than the struck point (vide, fig. 8B) is in negative direction with respect to positive impinging velocity of the load. This is also a mode effect It shows that the rod begins to vibrate soon after impact begins.

VERIFICATION OF THE THEORY

The values of γ_s, are required to use different expressions for displacement, pressure etc. (vide. Part I to III, Banerjee, 1966) The hammer is taken to be hard and it does not materially affect the results The $\eta_1 \sim \gamma_s$ and $\eta_2 \sim \gamma_s$, (vide. Part II, Banerjee, 1966) curves for particular struck points may be drawn as per eqn. 7, Pt. II (Banerjee, 1966) knowing k_1 and k_2 for the struck point Such a curve for $x = a = l/2$, i e. mid-pt , is shown for 'mass ratios', 3 71 and 3.29. (Fig. 5)

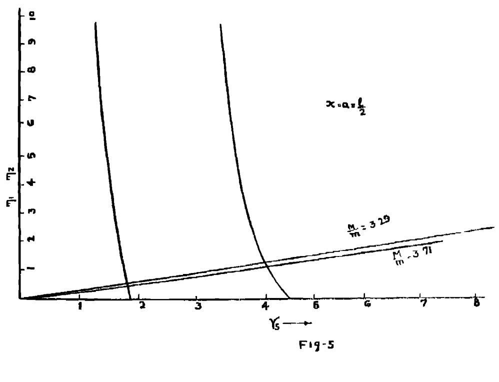


Fig-5

Duration of impact

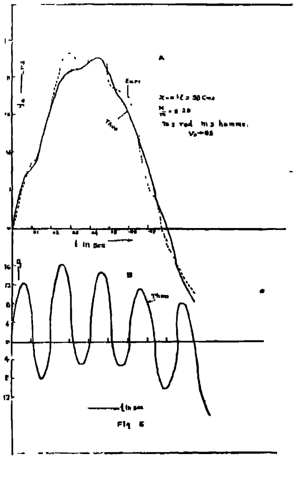
Thus it is found that the difference in the theoretical and experimental values of ϕ is very small and the experimental values are having higher magnitude than the theoretical ones, showing usefulness of the extension of the theory in light of Hertz's theory of impact (vide. Part III, Banerjee 1966). In applying Hertz's theory of impact, we have considered the case of cartilever A struck by Hammer C at 80 cms from fixed end (vide. fig. 8D) Only the first term of the pressure

cantilever load combination	Mass ratio	striking velocity cms /sec.	striking distance	no. of modes	Duration of impact (secs.) Theo. ϕ_o	Expt. ϕ	Difference $\phi_o - \phi$
A—C	3.29	65	a = 1	2	.011	.0154	—.004
A—C	3 29	94.75	a = 1	3	.0044	.0045	—.0001
A—C	3.29	65	a = 1/2	2	.0105	.0120	—.0015
A—D	3.29	65	a = 1	2	.011	.012	—.001
B—C	3 55	32 75	a = 1	2	016	.020	—.004
B—E	8.36	65	a = 1	2	.0145	.0175	—.0025

eqn. 5 (pt. III, Banerjee 1966) is considered as in this case the second term is very nearly zero and the higher terms are negligible. The variation of ϕ with v_0 is calculated as per eqn. 8 (part III). ϕ_0 is calculated in the usual way from pressure eqn. 37, in Pt. III. for $P = 0$, and its value in this case is .0695 sec. Both the experimental and theoretical variation of ϕ with v_0 are shown in Fig. 2, u_0 is taken as .434 cms. It is evident from these curves that the theory put forward (considering Hertz's theory of impact) agrees nicely with experiment.

Displacement of the struck point

Fig. 6 gives the time-displacement curves (Theo and Expt) for the struck point at the free-end corresponding to the case given by shadowgraph C (fig. 8).



The curves are identical upto about .014 sec. which is nearly the time of duration of impact. This is in agreement with the theory. While the theoretical curve is extended beyond the time of impact (first contact) with the help of corresponding equation upto the time not exceeding half the period of vibration of the cantilever, it is observed that the two curves fit in closely. The undulations

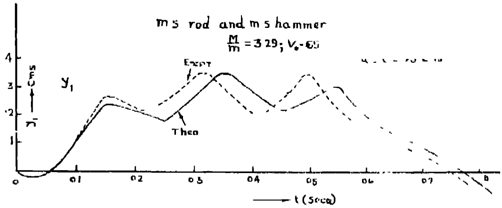


Fig = 7

(occurring almost in similar regions in the general body of respective curves) are more pronounced in the experimental curves. The difference between the two curves is due to sudden change in velocity suffered by the rod at the termination of a contact or separation, as anticipated in part III (Baneree, 1966). In the same Fig. 6, the theoretical pressure-time curve (in arbitrary scale) shows that at about .011 sec. after impact begins, pressure becomes zero. This time is therefore the duration of impact. The experimental value is .015 sec.

For striking distance, $x = a = 80$ cms (fig 8D), experimentally the time, measured for half the vibration of the bar is about 07 sec. and theoretically it is

Fig, 7

.0695 sec. This is a remarkable agreement. Further the amplitudes for experimental time-displacement curves for striking velocities of 32.75, 47.36 and 65 cms/sec. bear ratios 5 : 7.1 : 10 respectively, which are nearly the ratios among striking velocities. This is in full agreement of the theory. (Banerjee 1966, Pt. 11).

Displacement of any point other than the struck point

Fig. 7 shows the theoretical and experimental time-displacement curves for mid-pt. of the Cantilever-A, being struck by hammer–C at its free end with a velocity of 65 cms/sec (fig, 8B). The two curves are identical upto about .014 sec. which is almost the time of impact (first contact) It is noted that the two curves coincide in the region of negative displacement towards the earliest period of impact This is in complete agreement with the theory. Here also we have extended the theoretical curve beyond the time of first contact and find that the two curves are strikingly similar. Here also the difference between the two curves is due to after-impact effects, as discussed in Part III. (Banerjee, 1966).

Velocity of the hammer

striking distance in cms	Cantilever —hammer	striking velocity cms/sec	Velocity at the end of impact (cms/sec.)		Observation
			Theo	Expt.	
95	A – C	65	+31 68	+30 15	multiple contact
95	B – C	32 75	+10 24	+ 9 66	–do–
95	B—E	65	+ 7.5	+ 5 6	–do–
47 5	A—C	65	—14.5	—13.9	Impact ends

Here also the agreement between the theoretical and experimental values is excellent. Further works on the behaviour of the beam after impact is in progress and will be reported shortly.

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